Pion Velocity near the Chiral Phase Transition Point in the Vector Manifestation *

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Abstract

We study the pion velocity near the critical temperature T_c of chiral symmetry restoration in QCD. Using chiral perturbation theory based on hidden local symmetry (HLS) as an effective field theory, where the chiral symmetry restoration is realized as the vector manifestation (VM), we show that the pion velocity for $T \to T_c$ is close to the speed of light, $v_\pi(T) = 0.83 - 0.99$ which is at variance with the sigma model result which predicts $v_\pi(T_c) = 0$. Our prediction on $v_\pi(T_c)$ is compared with the value extracted from the recent RHIC data.

1 Introduction

It has been suggested by Cramer et al. [1] that the recent result by the STAR collaboration at RHIC [2] provides information on the pion velocity near the chiral phase transition in hot medium with the conclusion that the pions seen in Hanbury Brown-Twist interferometry are emitted from a chiral-symmetry restored phase [3]. In this note, we wish to interpret the result of the analysis by Cramer et al. in terms of the vector manifestation scenario of hidden local symmetry theory [4, 5].

The effective field theory based on the hidden local symmetry (HLS), which includes both pions and vector mesons as the dynamical degrees of freedom, implemented with the Wilsonian matching to determine the bare theory from the underlying QCD, leads to the vector manifestation (VM)

of chiral symmetry [4, 5] in which the massless vector meson becomes the chiral partner of the pion at the critical point [#1]. This picture provides a strong support for Brown-Rho scaling [6] which predicted that the mass of light-quark hadrons should drop in proportion to the quark condensate $\langle \bar{q}q \rangle$. By now there are several experimental indications that this scenario is a viable one. The earliest one was the enhancement of dielectron mass spectra below the ρ/ω resonance observed at CERN SPS [7] which has been satisfactorily explained by the dropping of the ρ meson mass [8] according to the Brown-Rho scaling [6]. This explanation however is not unique as there are alternative – but not necessarily unrelated – mechanisms that can equally well describe the presently available data [9]. A much more compelling evidence comes from the mass shift of the ω meson in nuclei measured by the KEK-PS E325 Experiment [10] and the CBELSA/TAPS Collaboration [11], and also from that of the ρ meson observed in the STAR experiment [12]. These are clean signals manifested in a "pristine" environment unencumbered by a plethora of "trivial" effects.

In this note, we focus on the pion velocity near the critical temperature and make a prediction based on the vector manifestation (VM). The pion velocity is one of the important observable quantities in heavy-ion collisions as it controls the pion propagation in medium through a dispersion relation. Our prediction for v_π is $v_\pi(T_c) = 0.83 - 0.99$, which we should stress, is at

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[#1] As studied in Ref. [5] in detail, the VM is defined only as a limit with bare parameters approaching the VM fixed point from the broken phase.

strong variance with the result obtained in sigma models [#2], i.e., $v_\pi(T_c) = 0$ [13]. We believe our result to be consistent with $v_\pi(T) = 0.65$ of Cramer *et al.* [1] extracted from the recent STAR data [2]. What distinguishes our approach from that of sigma models is the *intrinsic temperature and/or density dependence of the parameters* of the HLS Lagrangian, that results from integrating out the high energy modes (i.e., the quarks and gluons above the matching scale) *in medium* [14, 15]. It is this intrinsic temperature and/or density dependence – which causes Lorentz symmetry breaking – that plays the essential role for realizing the chiral symmetry restoration in a consistent way and underlies the Brown-Rho scaling.

2 Vector manifestation of chiral symmetry

In this section, we start with the HLS Lagrangian at leading order including the effects of Lorentz non-invariance. Then we present the conditions satisfied at the critical point.

The HLS theory is based on the $G_{\text{global}} \times H_{\text{local}}$ symmetry, where $G = SU(N_f)_{\text{L}} \times SU(N_f)_{\text{R}}$ is the chiral symmetry and $H = SU(N_f)_{\text{V}}$ is the HLS. The basic quantities are the HLS gauge boson V_μ and two matrix valued variables $\xi_{\text{L}}(x)$ and $\xi_{\text{R}}(x)$ which transform as $\xi_{\text{L,R}}(x) \to \xi'_{\text{L,R}}(x) = h(x)\xi_{\text{L,R}}(x)g^\dagger_{\text{L,R}}$, where $h(x) \in H_{\text{local}}$ and $g_{\text{L,R}} \in [SU(N_f)_{\text{L,R}}]_{\text{global}}$. These variables are parameterized as [#3] $\xi_{\text{L,R}}(x) = e^{i\sigma(x)/F^t_\sigma}e^{\mp i\pi(x)/F^t_\pi}$, where $\pi = \pi^a T_a$ denotes the pseudoscalar Nambu-Goldstone (NG) bosons associated with the spontaneous symmetry breaking of G_{global} chiral symmetry, and $\sigma = \sigma^a T_a$ denotes the NG bosons associated with the spontaneous breaking of H_{local}. This σ is absorbed into the HLS gauge boson through the Higgs mechanism, and then the vector meson acquires its mass. F^t_π and F^t_σ denote the temporal components of the decay constant of π and σ, respectively. The covariant derivative of ξ_L is given by

$$D_\mu \xi_L = \partial_\mu \xi_L - iV_\mu \xi_L + i\xi_L \mathcal{L}_\mu, \qquad (1)$$

[#2]By sigma models, we mean generically the chiral symmetry models that contain pions as the *only* relevant long-wavelength degrees of freedom.

[#3]The wave function renormalization constant of the pion field is given by the temporal component of the pion decay constant [16, 17]. Thus we normalize π and σ by F^t_π and F^t_σ respectively.

and the covariant derivative of ξ_R is obtained by the replacement of $\mathcal{L}_\mu$ with $\mathcal{R}_\mu$ in the above where V_μ is the gauge field of H_{local}, and $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$ are the external gauge fields introduced by gauging G_{global} symmetry. In terms of $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$, we define the external axial-vector and vector fields as $\mathcal{A}_\mu = (\mathcal{R}_\mu - \mathcal{L}_\mu)/2$ and $\mathcal{V}_\mu = (\mathcal{R}_\mu + \mathcal{L}_\mu)/2$.

In the HLS theory it is possible to perform the derivative expansion systematically [18, 19, 20, 5]. In the chiral perturbation theory (ChPT) with HLS, the vector meson mass is to be considered as small compared with the chiral symmetry breaking scale Λ_χ, by assigning $\mathcal{O}(p)$ to the HLS gauge coupling, $g \sim \mathcal{O}(p)$ [18, 19]. (For details of the ChPT with HLS, see Ref. [5].) The leading order Lagrangian with Lorentz non-invariance can be written as [15]

$$\mathcal{L} = \left[(F^t_\pi)^2 u_\mu u_\nu + F^t_\pi F^s_\pi \left(g_{\mu\nu} - u_\mu u_\nu \right) \right]$$
$$\times \text{tr} \left[\hat{\alpha}^\mu_\perp \hat{\alpha}^\nu_\perp \right]$$
$$+ \left[(F^t_\sigma)^2 u_\mu u_\nu + F^t_\sigma F^s_\sigma \left(g_{\mu\nu} - u_\mu u_\nu \right) \right]$$
$$\times \text{tr} \left[\hat{\alpha}^\mu_\parallel \hat{\alpha}^\nu_\parallel \right]$$
$$+ \left[-\frac{1}{g^2_L} u_\mu u_\alpha g_{\nu\beta} \right.$$
$$\left. - \frac{1}{2g^2_T} \left(g_{\mu\alpha} g_{\nu\beta} - 2u_\mu u_\alpha g_{\nu\beta} \right) \right]$$
$$\times \text{tr} \left[V^{\mu\nu} V^{\alpha\beta} \right] , \qquad (2)$$

where

$$\hat{\alpha}^\mu_{\perp,\parallel} = \frac{1}{2i}[D^\mu \xi_R \cdot \xi^\dagger_R \mp D^\mu \xi_L \cdot \xi^\dagger_L]. \qquad (3)$$

Here F^s_π denote the spatial pion decay constant and similarly F^s_σ for the σ. The rest frame of the medium is specified by $u^\mu = (1, \vec{0})$ and $V_{\mu\nu}$ is the field strength of V_μ. g_L and g_T correspond in medium to the HLS gauge coupling g. The parametric π and σ velocities are defined by [21]

$$V^2_\pi = F^s_\pi/F^t_\pi, \qquad V^2_\sigma = F^s_\sigma/F^t_\sigma. \qquad (4)$$

Now we approach the critical point of chiral symmetry restoration, which is characterized by the equality between the axial-vector and vector current correlators in QCD, $G_A - G_V \to 0$ for $T \to T_c$. This should hold even in the EFT side. In Ref. [15], it was shown that they are satisfied for any values of p_0 and $\bar{p}$ around the matching scale only if the following conditions are met: $(g_{L,\text{bare}}, g_{T,\text{bare}}, a^t_{\text{bare}}, a^s_{\text{bare}}) \to (0, 0, 1, 1)$ for

$T \to T_c$. This implies that at the bare level the longitudinal mode of the vector meson becomes the real NG boson and couples to the vector current correlator, while the transverse mode decouples. As shown in Ref. [16], $(g_L, a^t, a^s) = (0, 1, 1)$ is a fixed point of the RGEs and satisfied at any energy scale. Thus the VM condition is given by

$$(g_L, a^t, a^s) \to (0, 1, 1) \quad \text{for} \quad T \to T_c. \tag{5}$$

The vector meson mass is never generated at the critical temperature since the quantum correction to M_ρ^2 is proportional to g_L^2. Because of $g_L \to 0$, the transverse vector meson at the critical point, at any energy scale, decouples from the vector current correlator. The VM condition for a^t and a^s leads to the equality between the π and σ (i.e., longitudinal vector meson) velocities:

$$(V_\pi/V_\sigma)^4 = (F_\pi^s F_\sigma^t / F_\sigma^s F_\pi^t)^2 = a^t/a^s \overset{T \to T_c}{\to} 1. \tag{6}$$

This can be easily understood from the point of view of the VM since the longitudinal vector meson becomes the chiral partner of pion. We note that this condition $V_\sigma = V_\pi$ holds independently of the value of the bare pion velocity which is to be determined through the Wilsonian matching.

3 Pion velocity near the critical temperature

One possible way to determine the bare parameters is the Wilsonian matching which is done by matching the axial-vector and vector current correlators derived from the HLS with those by the operator product expansion (OPE) in QCD at the matching scale Λ [20]. The Wilsonian matching leads to the following conditions on the bare pion decay constants [22]:

$$
\frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s}{\Lambda^2} = \frac{1}{8\pi^2}\left[\left(1 + \frac{\alpha_s}{\pi}\right)\right.
$$
$$
+ \frac{2\pi^2}{3}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle_T}{\Lambda^4} + \pi^3\frac{1408}{27}\frac{\alpha_s\langle\bar{q}q\rangle_T^2}{\Lambda^6}\left.\right]
$$
$$
+ \frac{\pi^2}{15}\frac{T^4}{\Lambda^4}A_{4,2}^\pi - \frac{16\pi^4}{21}\frac{T^6}{\Lambda^6}A_{6,4}^\pi \equiv G_0,
$$
$$
\frac{F_{\pi,\text{bare}}^t F_{\pi,\text{bare}}^s (1 - V_{\text{bare}}^2)}{\Lambda^2} = \frac{32}{105}\pi^4\frac{T^6}{\Lambda^6}A_{6,4}^\pi,
$$
$$
\tag{7}
$$

where we use the dilute pion-gas approximation in order to evaluate the matrix element $\langle\mathcal{O}\rangle_T$ [23]

in the low temperature region. From these conditions, we obtain the following matching condition to determine the deviation of the bare pion velocity from the speed of light in the low temperature region:

$$\delta_{\text{bare}} \equiv 1 - V_{\pi,\text{bare}}^2 = \frac{1}{G_0}\frac{32}{105}\pi^4\frac{T^6}{\Lambda^6}A_4^\pi. \tag{8}$$

This implies that the intrinsic temperature dependence starts from the $\mathcal{O}(T^6)$ contribution.

As is discussed in Ref. [22], we should in principle evaluate the matrix elements in terms of QCD variables only in order for performing the Wilsonian matching, which is as yet unavailable from model-independent QCD calculations. Therefore, we make an estimation by extending the dilute gas approximation adopted in the QCD sum rule analysis in low temperature region to the critical temperature including all the light degrees of freedom expected in the VM. In the HLS/VM theory, both the longitudinal and transverse vector mesons become massless at the critical temperature. At the critical point, the longitudinal vector meson couples to the vector current whereas the transverse vector mesons decouple from the theory. Thus we assume that thermal fluctuations of the system are dominated near T_c not only by the pions but also by the longitudinal vector mesons. We evaluate the thermal matrix elements of the non-scalar operators in the OPE, by extending the thermal pion gas approximation employed in Ref. [23] to the longitudinal vector mesons that figure in our approach.

This is feasible since at the critical temperature, we expect the equality $A_4^\rho(T_c) = A_4^\pi(T_c)$ to hold as the massless longitudinal vector meson is the chiral partner of the pion in the VM. It should be noted that, although we use the dilute gas approximation, the treatment here is already beyond the low-temperature approximation because the contribution from vector meson is negligible in the low-temperature region. Since we treat the pion as a massless particle in the present analysis, it is reasonable to take $A_4^\pi(T) \simeq A_4^\pi(T = 0)$. We therefore use

$$A_4^\rho(T) \simeq A_4^\pi(T) \simeq A_4^\pi(T = 0) \quad \text{for} \quad T \simeq T_c. \tag{9}$$

Therefore from Eq. (8), we obtain the deviation δ_{bare} as

$$\delta_{\text{bare}} = 1 - V_{\pi,\text{bare}}^2 = \frac{1}{G_0}\frac{32}{105}\pi^4\frac{T^6}{\Lambda^6}\left[A_4^\pi + A_4^\rho\right]. \tag{10}$$

This is the matching condition to be used for determining the value of the bare pion velocity near the critical temperature. Let us make a rough estimate of δ_{bare}. For the range of matching scale ($\Lambda = 0.8 - 1.1\,\text{GeV}$), that of QCD scale ($\Lambda_{QCD} = 0.30 - 0.45\,\text{GeV}$) and critical temperature ($T_c = 0.15 - 0.20\,\text{GeV}$), we get

$$\delta_{\text{bare}}(T_c) = 0.0061 - 0.29\,. \tag{11}$$

Thus we obtain the *bare* pion velocity as $V_{\pi,\text{bare}}(T_c) = 0.83 - 0.99\,.$

We next consider the quantum and hadronic thermal corrections to the parametric pion velocity. It was proven in Ref. [16] that the pion velocity is protected from renormalization by the VM. In the following, we show that this can be understood in terms of chiral partners: Away from T_c, the pion velocity receives hadronic thermal correction of the form [24]:

$$
\begin{aligned}
v_\pi^2(T) &\simeq V_\pi^2 - N_f \frac{2\pi^2}{15} \frac{T^4}{(F_\pi^t)^2 M_\rho^2} \\
&\quad \text{for} \quad T < T_c,
\end{aligned} \tag{12}
$$

where the contribution of the massive σ (i.e., the longitudinal mode of massive vector meson) is suppressed by the Boltzmann factor $\exp[-M_\rho/T]$, and then only the pion loop contributes to the pion velocity. On the other hand, when we approach the critical temperature, the vector meson mass goes to zero due to the VM. Thus $\exp[-M_\rho/T]$ is no longer the suppression factor. As a result, the hadronic correction in the pion velocity is absent due to the exact cancelation between the contribution of pion and that of its chiral partner σ. Similarly the quantum correction generated from the pion loop is exactly canceled by that from the σ loop. Accordingly we conclude

$$v_\pi(T) = V_{\pi,\text{bare}}(T) \qquad \text{for} \quad T \to T_c, \tag{13}$$

i.e., *the pion velocity in the limit $T \to T_c$ receives neither hadronic nor quantum corrections due to the protection by the VM.* This implies that $(g_L, a^t, a^s, V_\pi) = (0, 1, 1, \text{any})$ forms a fixed line for four RGEs of g_L, a^t, a^s and V_π. When a point on this fixed line is selected through the matching procedure as explained in Ref. [22], that is to say when the value of $V_{\pi,\text{bare}}$ is fixed, the present result implies that the point does not move in a subspace of the parameters. Approaching the chiral symmetry restoration point, the physical pion velocity itself will flow into the fixed point. Finally

thanks to the non-renormalization property, i.e., $v_\pi(T_c) = V_{\pi,\text{bare}}(T_c)$ given in Eq. (13), we arrive at the physical pion velocity at the chiral restoration:

$$v_\pi(T_c) = 0.83 - 0.99\,, \tag{14}$$

close to the speed of light.

4 Conclusion

In this note, we studied, using the ChPT with HLS/VM, the pion velocity near the critical temperature. We exploited the non-renormalization property of the pion velocity to assure that it suffices to compute the *bare* pion velocity at the matching scale to arrive at the *physical* pion velocity at the chiral restoration temperature. We derived the matching condition on the *bare* pion velocity and found that the pion velocity near T_c is close to the speed of light, $v_\pi^{(\text{VM})}(T) = 0.83 - 0.99$ and definitely far from the zero velocity. This is in a stark contrast to the result obtained from the chiral theory [13], wherein only the pion figures as the relevant degree of freedom near T_c, namely, $v_\pi(T_c) = 0$. The drastic difference between the two approaches is not difficult to understand. In the HLS/VM approach, the ρ meson becomes light as T_c is approached from below and plays as important a role as the pion does. The effect of the massless vector meson cannot be approximated in chiral models by local operators in pseudoscalar fields.

By fitting the pion spectra observed by STAR at RHIC in terms of an optical potential that incorporates the dispersion relation of low-energy pions in nuclear matter, Cramer et al. deduced the in-medium pion velocity $v_\pi(T) = 0.65$. The authors interpreted this result as an evidence for the pions being emitted from the chiral-symmetry restored phase. No error bars have been assigned to this value, so it is difficult to make a clear-cut assessment of what that value implies. It seems however difficult to identify it with what has been predicted by sigma models, namely, $v_\pi = 0$. On the other hand, given that our prediction (14) based on HLS/VM is for the chiral limit, it seems reasonable to expect that the account of the explicit chiral symmetry breaking by quark masses would lower the velocity from (14), making it closer to the observed value, $v_\pi = 0.65$. Whether or not it signals the chirally restored phase as interpreted in [1, 3] is not clear. However as argued in [25], there

is an indication for a massless pion (in the same multplet with a scalar σ) *just above T_c* with its velocity close to 1, it seems logical that v_π stays near 1 – rather than near zero – as T_c is approached from below as well as from above.

Acknowledgments

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